EXAMPLE 3.2

In the circuit of Fig. 3.24, a zener diode with \( V_{ZK} = 3 \text{ V} \) is connected to a resistive circuit that tends to operate the zener in its reverse-biased region. From the circuit diagram alone, it is not evident whether sufficient voltage is developed across the zener to initiate reverse breakdown. Use the graphical technique to determine whether the zener is in its reverse-biased or reverse-breakdown region of operation.

**Solution**

The analysis parallels that of the pn junction diode circuit of Section 3.3.3. The zener, which is the only nonlinear element in the circuit, is temporarily disconnected, and the Thévenin equivalent of the rest of the circuit is found.

- **Find the open-circuit voltage of the resistive circuit with the zener disconnected**

  In this case the open-circuit voltage of the resistive circuit consisting of \( V_O \) and the three resistors is given by
  \[
  v_{Th} = V_O \frac{R_2}{R_1 + R_2} = 12 \text{ V} \frac{1 \text{ k\Omega}}{1 \text{ k\Omega} + 1 \text{ k\Omega}} = 6 \text{ V}
  \]  
  (3.27)
  Note that \( R_3 \) does not appear in the expression for \( v_{Th} \) because no current flows through it when the zener is disconnected.

- **Find the Thévenin resistance of the resistive circuit seen by the zener**

  The Thévenin equivalent resistance seen by the zener at terminals \( X - X' \) can be found by setting the voltage source \( V_O \) to zero (short circuit):
  \[
  R_{Th} = R_1 R_2 + R_3 = 1 \text{ k\Omega} \parallel 1 \text{ k\Omega} + 500 \text{ \Omega} = 1 \text{ k\Omega}
  \]  
  (3.28)
  Note that although the presence of \( R_3 \) does not affect the open-circuit voltage, it *does* contribute directly to the value of \( R_{Th} \).

- **Find the short-circuit current of the resistive circuit by calculating the ratio of \( v_{Th} \) to \( R_{Th} \)**

  Finally, \( i_{SC} \) is found from
  \[
  i_{SC} = \frac{v_{Th}}{R_{Th}} = \frac{6 \text{ V}}{1 \text{ k\Omega}} = 6 \text{ mA}
  \]  
  (3.29)

- **Find the operating point of the zener**

  In order to find the values of \( V_Z \) and \( I_Z \), we need only reconnect the zener to the Thévenin equivalent circuit and utilize the graphical solution technique. In Fig. 3.24, the zener variables \( V_Z \) and \( I_Z \) are of opposite polarity to those of the Thévenin circuit terminal variables \( V_X \) and \( I_X \). If the \( v-i \) characteristics of both the zener and the Thévenin circuit are to be plotted on the same set of axes, the \( v-i \) curve of one of them must be inverted. This technique was used to analyze the diode circuit of Fig. 3.21 in Section 3.3.3.
Zener Diode I-V Characteristic
Is Inverted in $V_X - i_X$ Coordinates System

BECAUSE

Current Flow Reference Direction for $i_X$
IS OPPOSITE OF
Current Flow Reference Direction for $i_Z$
AND
Voltage Drop Reference Direction for $V_X$
IS OPPOSITE OF
Current Flow Reference Direction for $V_Z$

\[ i_X = -i_Z \]

\[ v_X = -v_Z \]

**Figure 3.25** Load line of the resistive circuit of Fig. 3.24 superimposed on the inverted $v-i$ characteristic of a zener diode with $V_{ZK} = 3 \text{ V}$.

The graphical inversion of the zener's $v-i$ characteristic about both the $v$-axis and $i$-axis is performed in Fig. 3.25. From these plots, the terminal variables of the Thévenin circuit are seen to be

\[ v_X \approx V_{ZK} = 3 \text{ V} \]
\[ i_X \approx 3 \text{ mA} \]

(3 V at the output terminals) \hspace{1cm} (3 mA flowing out of the Thévenin circuit) \hspace{1cm} (3.30) \hspace{1cm} (3.31)

The voltage applied to the zener, as defined by the zener's own variable $v_Z$, becomes

\[ v_Z = -v_X = -V_{ZK} = -3 \text{ V} \]

(3.32)

Similarly, the zener current becomes

\[ i_Z = -i_X = -3 \text{ mA} \]

(3 mA flowing in the reverse direction through the zener) \hspace{1cm} (3.33)

For this circuit, the operating point will be located over the "knee" of the zener (i.e., in the reverse-breakdown region) as long as $v_{th} > V_{ZK}$, where $V_{ZK}$ is a positive number. Conversely, if $v_{th} < V_{ZK}$, the voltage appearing at the zener's terminals will be of insufficient magnitude to initiate reverse breakdown, and $i_Z$ will fall to zero.

**EXERCISE 3.13**

Find the voltage across the zener in Fig. 3.24 if $V_O$ is changed from 12 to 16 V. The zener has a $V_{ZK}$ of 3 V.  

**Answer:** $v_Z = -3 \text{ V}$

3.14 Find the operating point of the zener in Fig. 3.24 if $V_O$ is changed from 12 to 6 V. The zener has a $V_{ZK}$ of 3 V.  

**Answer:** $v_Z = -3 \text{ V}; i_Z = 0$

3.15 Find the operating point of the zener in Fig. 3.24 if $V_O$ is changed from 12 to 4 V. The zener has a $V_{ZK}$ of 3 V.  

**Answer:** $v_Z = -2 \text{ V}; i_Z = 0$