Exercise 4.9

(a) $V_{BE} = -0.2$ V and $V_{CE} = 5$ V, because we have $V_{BE} < 0.5$, the transistor is in cutoff.

(b) $I_B = 50 \mu A$ and $I_C = 2$ mA, because we have $I_C < \beta I_B$ the transistor is in saturation.

(c) $V_{CE} = 5$ V and $I_B = 50 \mu A$, because we have $V_{CE} > 0.2$ and $I_B > 0$, the transistor is in the active region.

Exercise 4.10

(a) Let us assume operation in the active region. Then we have $I_B = (V_{CC} - 0.7)/R_B = 71.5 \mu A$, $I_C = \beta I_B = 3.575$ mA, and $V_{CE} = V_{CC} - R_C I_C = 11.4$ V. Because we found $V_{CE} > 0.2$ V, the active-region assumption is valid and the results are correct.

(b) Again let us assume operation in the active region. Then we have $I_B = (V_{CC} - 0.7)/R_B = 71.5 \mu A$, $I_C = \beta I_B = 17.9$ mA, and $V_{CE} = V_{CC} - R_C I_C = -2.9$ V. Because we found $V_{CE} < 0.2$ V, the active-region assumption is invalid, and the results are not correct.

Therefore let us assume operation in saturation. Then we have $I_B = (V_{CC} - 0.7)/R_B = 71.5 \mu A$, $I_C = (V_{CC} - 0.2)/R_C = 14.8$ mA. Because we have $\beta I_B > I_C$ the saturation-region assumption is valid.

Exercise 4.11

The load-line equation is

$$V_{CC} = R_C I_C + V_{CE} \quad \text{or} \quad 20 = 5000 I_C + V_{CE}$$
A plot of the load line is:

\[ I_C \]

\[ 4 \text{mA} \]
\[ 2 \text{mA} \]
\[ \text{middle of load line} \]
\[ 10 \text{V} \]
\[ 20 \text{V} \]

(a) \[ I_B = I_C / \beta = (2 \text{ mA}) / 100 = 20 \text{ µA} \]

\[ R_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{20 - 0.7}{20 \times 10^{-6}} = 965 \text{ kΩ} \]

(b) \[ I_B = I_C / \beta = (2 \text{ mA}) / 300 = 6.67 \text{ µA} \]

\[ R_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{20 - 0.7}{6.67 \times 10^{-6}} = 2.9 \text{ MΩ} \]

Exercise 4.12

We assume operation in the active region. Then we have

\[ I_C = \beta I_B = 0.965 \text{ mA} \]

\[ V_{CE} = -20 + R_C I_C = -10.35 \text{ V} \]

Because \( V_{CE} < -0.2 \text{ V} \), the transistor is in fact operating in the active region and the problem is solved.

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(b) As in part (a) we have \( I_B = 19.3 \, \mu A \). We start by assuming operation in the active region resulting in

\[
I_C = \beta I_B = 2.90 \, mA
\]

\[
V_{CE} = -20 + R_C I_C = 9 \, V
\]

Because \( V_{CE} > -0.2 \, V \), the active region assumption is not valid. Therefore assume operation in saturation, in which case we have

\[
I_B = \frac{V_{CC} + V_{BE}}{R_B} = \frac{20 - 0.7}{1 \, \Omega} = 19.3 \, \mu A
\]

\[
V_{CE} = -0.2 \, V
\]

\[
I_C = \frac{V_{CC} - 0.2}{R_C} = 1.98 \, mA
\]

Then because \( \beta I_B > I_C \) the transistor is operating in saturation, and the problem is solved.

**Exercise 4.13**

\[
R_1 = 100 \, k\Omega \\
R_2 = 50 \, k\Omega
\]

\[
R_B = \frac{1}{1/R_1 + 1/R_2} = 33.3 \, k\Omega
\]

\[
V_B = V_{CC} \frac{R_2}{R_1 + R_2} = 5 \, V
\]

\[
I_B = \frac{V_B - V_{BE}}{R_B + (\beta + 1)R_E} = \frac{5 - 0.7}{33.3k + (\beta+1)1k}
\]

\[
I_C = \beta I_B
\]

\[
I_E = I_C + I_B
\]

\[
V_{CE} = V_{CC} - R_C I_C - R_E I_E
\]

<table>
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<tr>
<th>( \beta )</th>
<th>( I_B ) (( \mu A ))</th>
<th>( I_C ) (mA)</th>
<th>( I_E ) (mA)</th>
<th>( V_{CE} ) (V)</th>
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<td>3.20</td>
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<tr>
<td>300</td>
<td>12.9</td>
<td>3.86</td>
<td>3.87</td>
<td>7.27</td>
</tr>
</tbody>
</table>

In Example 4.7 the ratio of the collector currents is 4.24/4.12 = 1.029. For the higher resistor values in this exercise the ratio is 3.86/3.20 = 1.21. In general higher resistance values in the four-resistor bias circuit lead to
From the equivalent circuit we can write: $v_{in} = -r_\pi i_b$ and $v_o = -\beta i_b R'_L$. Dividing the respective sides of these equations we obtain:

$$A_v = \frac{\beta R'_L}{r_\pi}$$

Writing a current equation at the input terminal we have:

$$i_{in} = \frac{v_{in}}{R_E} - (\beta + 1)i_b$$

Then we substitute $i_b = -\frac{v_{in}}{r_\pi}$ and rearrange to obtain:

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{1/R_E + (\beta + 1)/r_\pi}$$

The equivalent circuit for determining the output impedance is:

From the circuit we can write:

$$\frac{v_1}{R_s} + \frac{v_1}{R_E} = (\beta + 1)i_b \quad \text{and} \quad v_1 = -r_\pi i_b$$
Using the second equation to substitute into the first, we obtain $i_b = 0$. Thus the controlled source $\beta i_b$ is an open circuit, and we have

$$R_o = \frac{V_x}{i_x} = R_C$$

**Exercise 4.23**

The dc circuit is:

![Diagram showing circuit components](image)

$$R_B = R_1 || R_2 = 33.3 \text{ k}\Omega \quad V_B = \frac{V_{CC} R_2}{(R_1 + R_2)} = 5 \text{ V}$$

$$I_{BQ} = \frac{V_B - V_{BEQ}}{R_B + (\beta + 1)R_E} = 7.99 \mu\text{A} \quad I_{CQ} = \beta I_{BQ} = 0.799 \text{ mA}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = 3254 \Omega \quad R'_L = R_C || R_L = 833 \Omega$$

$$A_V = \frac{\beta R'_L}{r_\pi} = 25.6 \quad R_i = R_E || \left[ r_\pi / (\beta + 1) \right] = 32.0 \Omega$$

$$R_o = R_C = 5 \text{ k}\Omega \quad A_i = A_V R_i / R_L = 0.819 \quad G = A_V A_i = 21.0$$

**Exercise 4.24**

From the equivalent circuit shown in Figure 4.40 in the book we can write:

$$\frac{V_o}{R'_L} + \frac{V_o - V_{in}}{R_B} + \beta i_D = 0$$

Then using $i_D = \frac{V_{in}}{r_\pi}$ to substitute for $i_D$ and rearranging the resulting equation we obtain:

$$A_V = \frac{V_o}{V_{in}} = \frac{R'_L (r_\pi - \beta R_B)}{r_\pi (R'_L + R_B)}$$
For $v_{in} = +1$ we have $v_{GS} = 4$ and the instantaneous operating point is A. Similarly for $v_{in} = -1$ we have $v_{GS} = 2$ V and the instantaneous operating point is at B. We find $V_{DSQ} \approx 11$ V, $V_{DSmin} \approx 6$ V, $V_{DSmax} \approx 14$ V.

**Exercise 5.4**

The analysis is similar to Example 5.3 in the book.

$$K = \left(\frac{W}{L}\right)KP = 1 \text{ mA/V}^2$$
\[ V_G = V_{DD} \frac{R_2}{R_1 + R_2} = 20 \frac{1}{(1.5 + 1)} = 8 \text{ V} \]

\[ V_{GSQ}^2 + \left( \frac{1}{R_{SK}} - 2V_{to} \right)V_{GSQ} + (V_{to})^2 - \frac{V_G}{R_{SK}} = 0 \]

After values are substituted, we have

\[ V_{GSQ}^2 - 3.583V_{GSQ} + 0.6667 = 0 \]

Solving we find \( V_{GSQ} = 3.39 \text{ V} \). (The second root is extraneous and should be discarded.) Then we have

\[ I_{DQ} = K(V_{GSQ} - V_{to})^2 = 1.92 \text{ mA} \]

\[ V_{DSQ} = V_{DD} - (R_D + R_S)I_{DQ} = 10.8 \text{ V} \]

**Exercise 5.5**

We should choose \( R_D = 0 \) for a source follower. Many values will work for the other resistors. A reasonable set of values is \( R_S = 3.9 \text{ k}\Omega \), \( R_1 = 1 \text{ M}\Omega \), and \( R_2 = 2 \text{ M}\Omega \). These values yield \( I_{DQ} = 1.98 \text{ mA} \) and \( V_{DSQ} = 7.26 \text{ V} \). Use SPICE to check that your design provides a Q-point close to the desired value.

**Exercise 5.6**

From Figure 5.24 at an operating point defined by \( V_{GSQ} = 2.5 \text{ V} \) and \( V_{DSQ} = 6 \text{ V} \), we have

\[ g_m = \frac{\Delta i_D}{\Delta V_{GS}} = \frac{(4.4 - 1.1) \text{ mA}}{1 \text{ V}} = 3.3 \text{ mS} \]

\[ 1/r_d = \frac{\Delta i_D}{\Delta V_{DS}} = \frac{(2.9 - 2.3) \text{ mA}}{(14 - 2) \text{ V}} = 0.05 \times 10^{-3} \]

Taking the reciprocal, we find \( r_d = 20 \text{ k}\Omega \)
\[ 11 = V_{DC} + V_{2m} \]

and at \( t = 0.75 \text{ ms} \) we have:

\[ 16 = V_{DC} - V_{1m} - V_{2m} \]

Solving the previous three equations we have \( V_{DC} = 10.5 \text{ V} \), \( V_{2m} = 0.5 \text{ V} \) and \( V_{1m} = -6 \text{ V} \). Thus the percentage second-harmonic distortion is \( \frac{|V_{2m}|}{|V_{1m}|} \times 100\% = 8.33\% \).

**Problem 5.21**

First, we use Equation 5.16 to compute

\[ V_G = \frac{V_{DD}}{R_1 + R_2} = 5 \text{ V} \]

As in Example 5.3, we need to solve:

\[ \frac{V^2}{V_{GSQ}} + \left( \frac{1}{R_S K} - 2V_{to} \right) V_{GSQ} + (V_{to})^2 - \frac{V_G}{R_S K} = 0 \]

Substituting values, we have

\[ V^2_{GSQ} - 1.1489V_{GSQ} - 3.2553 = 0 \]

The roots are \( V_{GSQ} = 2.4679 \text{ V} \) and \(-1.319 \text{ V} \). The correct root is \( V_{GSQ} = 2.4679 \text{ V} \) which yields \( I_{DQ} = K(V_{GSQ} - V_{to})^2 = 0.5387 \text{ mA} \).

Finally we have \( V_{DSQ} = V_{DD} - R_{DQ}^T - R_S I_{DQ} = 9.936 \text{ V} \).

**Problem 5.22**

For this circuit we can write

\[ V_{GSQ} = 15 - I_{DQ} R_S \]

Assuming operation in saturation, we have

\[ I_{DQ} = K(V_{GSQ} - V_{to})^2 \]

using the first equation to substitute into the second equation we have

\[ I_{DQ} = K(15 - I_{DQ} R_S - V_{to})^2 = 0.25(14 - 3I_{DQ})^2 \]
where we have assumed that \( I_{DQ} \) is in mA. Rearranging we have

\[
I_{DQ}^2 - 9.777I_{DQ} + 21.777 = 0
\]

The correct root is the smaller one which is \( I_{DQ} = 3.432 \) mA. Then we have \( V_{DSQ} = 30 - R_DI_{DQ} - R_SI_{DQ} = 16.27 \) V.

**Problem 5.23**

Assuming that the MOSFET is in saturation, we have

\[
V_{GSQ} = 10 - I_{DQ}
\]

\[
I_{DQ} = K(V_{GSQ} - V_to)^2
\]

where we have assumed that \( I_{DQ} \) and \( K \) are in mA and mA/V\(^2\) respectively.

(a) Using the second equation to substitute in the first, substituting values and rearranging, we have

\[
V_{GSQ}^2 - 7V_{GSQ} + 6 = 0
\]

which yields

\[
V_{GSQ} = 6 \text{ V}
\]

(The other root, \( V_{GSQ} = 1 \) V, is extraneous.)

\[
I_{DQ} = 4 \text{ mA}
\]

\[
V_{DSQ} = 10 - 2I_{DQ} = 12 \text{ V}
\]

(b) Similarly we have

\[
V_{GSQ}^2 - 3.5V_{GSQ} - 1 = 0
\]

\[
V_{GSQ} = 3.765 \text{ V}
\]

\[
I_{DQ} = 6.234 \text{ mA}
\]

\[
V_{DSQ} = 20 - 2I_{DQ} = 7.53
\]

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Problem 5.26

We have \( V_G = V_{GSQ} = \frac{10R_2}{R_1 + R_2} = 2.5 \text{ V} \). Then we have
\[
I_DQ = K(V_{GSQ} - V_{to})^2 = 0.5625 \text{ mA}.
\]
\( V_{DSQ} = V_{DD} - R_D I_{DQ} = 4.375 \text{ V} \).

Problem 5.27

We have \( V_{GSQ} = V_{DSQ} = V_{DD} - R_D I_{DQ} \). Then substituting \( I_{DQ} = K(V_{GSQ} - V_{to})^2 \), we have
\[
V_{GSQ} = V_{DD} - R_D K(V_{GSQ} - V_{to})^2
\]
Substituting values and rearranging, we have
\[
V_{GSQ}^2 + 2V_{GSQ} - 39 = 0
\]
Solving we determine that \( V_{GSQ} = 5.325 \text{ V} \) and then we have \( I_{DQ} = K(V_{GSQ} - V_{to})^2 = 4.675 \text{ mA} \).

Problem 5.28

See Figure 5.23 in the book.

Problem 5.29

\[
q_m = \frac{\delta i_D}{\delta v_{GS}} \bigg|_{Q-point}
\]
\[
1/r_D = \frac{\delta i_D}{\delta v_{DS}} \bigg|_{Q-point}
\]

Problem 5.30

For \( \lambda = 0 \) the drain characteristics are horizontal in the saturation region and \( r_D = \infty \).
Problem 5.37

See Figure 5.25 in the book.

Problem 5.38

(a)

\[
\begin{align*}
V(t) & \quad \rightarrow \quad R \quad \rightarrow \quad I_{in} \\
& \quad \rightarrow \quad R_f \quad \rightarrow \quad I_{in} \\
& \quad \rightarrow \quad V_{in} \quad \rightarrow \quad V_{gs} \\
& \quad \rightarrow \quad g_m V_{gs} \quad \rightarrow \quad R_D \quad \rightarrow \quad V_x \\
& \quad \rightarrow \quad R_L' \\
& \quad \rightarrow \quad V_{in} \quad \rightarrow \quad g_m V_{gs} \quad \rightarrow \quad R_D \quad \rightarrow \quad V_x \\
& \quad \rightarrow \quad R_L' \\
\end{align*}
\]

(b) \[v_o = R'_L (i_{in} - g_m v_{in}) \]
\[i_{in} = (v_{in} - v_o)/R_f \]
\[A_v = \frac{v_o}{v_{in}} = \frac{R'_L - g_m R'_L R_f}{R'_L + R_f} \]
\[R_{in} = \frac{v_{in}}{i_{in}} = \frac{R_f}{1 - A_v} \]

The circuit used to determine output impedance is:

\[
\begin{align*}
& \quad \rightarrow \quad R \quad \rightarrow \quad R_f \\
& \quad \rightarrow \quad V_{gs} \quad \rightarrow \quad V_x \\
& \quad \rightarrow \quad g_m V_{gs} \quad \rightarrow \quad R_D \quad \rightarrow \quad V_x \\
& \quad \rightarrow \quad R_L' \\
\end{align*}
\]

We define \( R'_D = R_D \parallel (R + R_f) \)
\[v_{gs} = v_x \frac{R}{R + R_f} \]
\[i_x = \frac{v_x}{R'_D} + g_m v_{gs} \]
\[ R_0 = \frac{V_X}{I_X} = \frac{1}{\frac{1}{R_D'} + \frac{1}{R_f} + \frac{1}{R}} \]

(c) The dc circuit is:

\[ \begin{align*}
& \uparrow V_{DD} = +20 \\
& R_D = 3 \, k\Omega \\
& R_f \quad \text{in parallel with} \quad R_D \\
\end{align*} \]

\[ V_{GSQ} = V_{DSQ} \quad I_{DQ} = K(V_{DSQ} - V_{to})^2 \quad I_{DQ} = (V_{DD} - V_{DSQ})/R_D \]

Using the above equations we obtain

\[ 3V_{DSQ}^2 - 29V_{DSQ} + 55 = 0 \]

\[ V_{DSQ} = 7.08 \, V \text{ and } I_{DQ} = 4.31 \, mA \]

\[ g_m = g_m = \frac{\delta i_d}{\delta v_{gs}} \text{ Q-point} \]

\[ = 2K(V_{GSQ} - V_{to}) = 4.16 \times 10^{-3} \, S \]

(d) \[ R_L' = R_D || R_L = 2.31 \, k\Omega \]

\[ A_V = -9.37 \]

\[ R_{in} = 9.64 \, k\Omega \]

\[ R_O = 414 \, \Omega \]

(e) \[ v_o(t) = v(t) \frac{R_{in}}{R + R_{in}} A_V = -0.164\sin(2000\pi t) \]

(f) This is an inverting amplifier that has a very low input impedance compared to many other FET amplifiers.

**Problem 5.39**

Referring to the circuit shown in Figure P5.39, we have

\[ V_{GSQ} = V_{DSQ} \quad I_{DQ} = K(V_{GSQ} - V_{to})^2 \quad I_{DQ} = (V_{DD} - V_{DSQ})/R_D \]