Op Amp for profit and fun

Simplest Model

\[ V_{out} = A \left( V_p - V_N \right) \] as long as \(-V_{EE} < V_{out} < V_{CC}\) [linear Region]

Ideal: \( R_{in} = \infty \Rightarrow i_p = 0, i_N = 0 \)

\[ R_o = 0 \]

\[ A = \infty \] (very large)

Saturation

If \( V_{out} = A \left( V_p - V_N \right) > V_{CC} \) then \( V_{out} = V_{CC} \)

If \( V_{out} = A \left( V_p - V_N \right) < -V_{EE} \) then \( V_{out} = -V_{EE} \)

We can have a large number of useful circuits using Op Amps, with important functions.

In the following we assume \( A \) is very large but finite, then we solve and find the circuit assuming operation in linear region. For each result we check the inequalities and if they don't hold we know Op Amp is in saturation.
Example of useful Circuits using OpAm

Example 1  Voltage Follower

1) Node o isshorted to node N  \( \Rightarrow V_o = V_N \)

2) Ideal Op Amp  \( \Rightarrow i_p = 0, i_N = 0 \)

3) Assume Op Amp in Linear Region  \( \Rightarrow V_o = A(V_p - V_N) \)

3) Circuit  \( \Rightarrow V_p = V_{in} \)

Put these together

\[ V_o = A(V_p - V_N) \Rightarrow V_o = A(V_{in} - V_o) \]

\[ \Rightarrow V_o = A \frac{V_{in}}{A+1} - A \frac{V_o}{A+1} \Rightarrow (A+1)V_o = AV_{in} \]

\[ \Rightarrow V_o = A \frac{V_{in}}{A+1} \]

\[ \Rightarrow V_o = V_{in} \text{ as } A \to \infty \]

Voltage Follower

Now let's find out  \( V_p - V_N \)

\[ V_p - V_N = V_{in} - V_o = V_{in} - \frac{A}{A+1} V_{in} \]

\[ V_p - V_N = \left( \frac{1}{A+1} \right) V_{in} \]

\[ \text{as } A \to \infty \]

\[ V_p - V_N = 0 \]
Notice if we put a load \( R_L \) across
the output we get the same result
i.e. \( V_0 = V_{in} \) and \( I_0 = \frac{V_0}{R_L} \)

From previous discussion \( I_{in} = I_p = 0 \)
\& \( I_N = 0 \)
so
so who supplies the current to the load? \( I_o \)
if \( V_{in} \) is not? Answer the \( V_{cc} \)

power supplies

Note: the amp saturates at \( V_{cc} - V_{EE} \)

to satisfy the assumption of linear
regime we must have

\( -V_{EE} < V_{in} < V_{EE} \)
\( V_0 = V_{in} \)
\( -V_{EE} < V_{in} < V_{cc} \)

This means that \( V_{in} \) must be limited
to the range between \( -V_{EE} \) to \( V_{cc} \)

Satisfaction: Let's suppose \( V_{in} > V_{cc} \)
then \( V_{out} \neq A(V_p - V_N) \) instead \( V_{out} = V_{cc} \)

Working backwards we see \( V_N = V_{cc} \) also

\( (V_p - V_{N}) = V_{in} - V_{cc} > 0 \) that is \( V_p - V_N \neq 0 \)

if \( V_{in} < -V_{EE} \) then \( V_0 = -V_{EE} \) \( V_{in} \) in Linear Regime
We saw \( V_p - V_N = 0 \)
Application of Voltage follower

1. Voltage divider:

Suppose we have a 12V source and we want to get 8V output for a load to apply to a load. We use a voltage divider:

\[ V_{out} = \frac{2R}{2R+R} V_{in} \]

\[ V_{out} = \frac{2 \times 12R}{3R} = 8V \text{ as desired} \]

Now comes the problem when we put a load:

Let \( R_L = R \) ⇒ 12V (the voltage to the load is)

\[ V_{load} = \frac{2R}{2R+R} \times 12 \]

\[ 2R + R = \frac{2R}{2R+R} \Rightarrow 2R = \frac{2R}{2R+R} = \frac{2}{3} R \]

\[ V_{load} = \frac{2R}{2R+R} \times 12 = \frac{2}{3} \times 12 = 4.8V \text{ not 9V!} \]

\[ L = \frac{4.8}{R_L} = \frac{4.8}{R} \]

This reduction in available voltage is called the loading effect. (Note: The load draws current which messes up the voltage divider)
Let's try $R_L = 8R$

\[ V_{\text{load}} = V_{\text{out}} \]

\[ 2 \times 8 \quad R = 10.6R \quad R \]

\[ \frac{2 + 8}{2+8} \]

Voltage divider: 12V

\[ V_{\text{out}} = \frac{10.6R}{2.6R} \times 12 = 7.38 \text{ V} < 9 \text{ V} \]

\[ L_L = \frac{7.38}{2.6R} \]

The voltage appearing at the load is smaller than what the design called for as dependent on the load resistance.

Now let put a voltage follower between source and voltage divider and the load.

\[ 12 + \quad L \]

\[ 2R \]
\( V_0 = V_N \)
\( V_0 = A \left( V_p - V_N \right) \)

Ideal Op Amp has \( R_{in} = \infty \Rightarrow i_p = 0 \)

\( \therefore i_s = \frac{12}{R+2R} = \frac{4}{R} \quad \text{Amp} \)
\( V_p = \frac{2R}{3} \times i_s = \frac{2 \times 12}{3R} = 8V \)

\( V_{out} = A \left( V_p - V_N \right) \)
\( V_{out} = A \left( 8 - V_{out} \right) \)
\( \therefore V_{out} = A \left( 8 \right) - A V_{out} \)
\( \therefore (A+1)V_{out} = 8A \)
\( \therefore V_{out} = \frac{8A}{A+1} \rightarrow 8 \)

The voltage for the load is independent of \( R_L \), w/o the op-amp voltage follower.

The role of Op Amp

Current is drawn from voltage devices.

The role of the voltage follower is to supply the current that load needs.
Example 3: Super diode

Consider the simple diode circuit below:

\[ I_D = I_0 \left( e^{\frac{V_D}{V_T}} - 1 \right) \]

\[ V_T = \frac{kT}{q} \approx 25 \text{ mV} \]

\[ V_D = \frac{V_T}{10} \] @ Room T

To eliminate the offset, we put an OpAmp between the source and terminals as follows:

\[ V_N = V_0 \]

\[ \frac{V}{R} \]

\[ V_D = V_I - V_0 \]
First assume $V > 0$ then we will show that the diode is conducting.

Assume $V_{in}$ is not too large and Op Amp is operating in linear region. In that case

$$V_p = A(V_{in} - V_D)$$

$$V_o = V_{in} - V_D$$

$$V_o = A(V_{in} - V_D)$$

$$AV_o + V_o = AV_{in} - V_D$$

$$V_o = \frac{AV_{in} - V_D}{A+1}$$

When $V_{in}$ is very large

$$V_o \approx V_{in}$$

as long as $V_{in} < V_C$

$$V_o = A(V_{in} - V_D) = A\frac{V_{in}}{A+1} + A\frac{V_D}{A+1}$$

$$V_1 = A(V_{in} + AV_{in}) - \frac{A}{A+1} V_D$$

For typical $V_{in}$ and $A$

$$V_D \ll AV_{in}$$

$A$ is very large.

\[ V_1 = A(V_{in} + AV_{in}) - \frac{A}{A+1} V_D \]
Now let's test our assumption that $V_D > 0 \Rightarrow V_i - V_o > 0 \Rightarrow V_i > V_o$

$\therefore V_i > V_i > 0 \text{ consistent}$

Now suppose $V_{in} < 0$

Then $V_o = \frac{-A V_{in}}{A+1}$

Now assume $R$ is a diode so in the off state (reverse bias)

Then circuit becomes

But what about $V_i$

If we can't use $V_i = A (V_D - V_n)$ because $R$ of amp is saturated with $V_{in} < 0 \Rightarrow V_i = -V_{EE} < 0$
Example 2: Non Inverting Amp.

\[ V_p = 0 \]

\[ i = 0 \]

\[ V_n = 0 \]

\[ V_{in} \]

\[ V_o \]

\[ V_p \]

\[ V_i \]

\[ V_{in} \]

\[ V_{o} \]

\[ V_{o} \]

\[ V_{o} \]

\[ V_{o} \]

\[ V_{o} \]

\[ V_{o} \]

\[ V_{o} \]

\[ V_{o} \]

Assume operation in linear region.

Then

\[ V_o = A(V_o - V_n) \]

\[ V_o = V_{in} \]

\[ i = \frac{V_o}{R_1 + R_2} \]

\[ V_n = \frac{R_2}{R_1 + R_2} i = \frac{R_2}{R_1 + R_2} V_o \]

\[ V_o = A(V_{in} - \frac{R_2}{R_1 + R_2} V_o) \]

\[ V_o(1 + \frac{R_2}{R_1 + R_2}) = AV_{in} \]

\[ V_o = \frac{AV_{in}}{1 + \frac{R_2}{R_1 + R_2}} \]

\[ N_o = \frac{V_{in}}{A V_{in}} \]

\[ N_o = \frac{1 + \frac{R_2}{R_1 + R_2}}{A} \]

\[ \Rightarrow |V_o| > |V_{in}| \]

As \( A \to \infty \)

\[ \Rightarrow \]

\[ V_o = \frac{R_2 + R_1}{R_2} V_{in} \]
\[ V_0 = \frac{R_2 + R_1}{R_2} V_{in} \text{ as long as } -\frac{V}{EE} < V < V_{cc} \]

To avoid saturation, \[ -\frac{V}{EE} < \frac{R_2 + R_1}{R_2} V_{in} < V_{cc} \]

\[ -\frac{R_2}{EE (R_2 + R_1)} < V_{in} < \frac{R_2}{R_1 + R_2} \]

Let's see what the value of \( V_P - V_N \) is in linear region

\[ V_P = V_{in}, \quad V_N = \frac{R_2}{R_2 + R_1} V_0, \quad V_0 = \frac{AV_{in}}{1 + \frac{R_2 A}{R_1 + R_2}} \]

\[ V_P - V_N = V_{in} - \frac{R_2}{R_2 + R_1} \times \frac{AV_{in}}{1 + \frac{R_2 A}{R_1 + R_2}} \]

\[ = V_{in} - \frac{R_2 A V_{in}}{(R_1 + R_2) + R_2 A} \]

\[ \frac{V_P - V_N}{V_{in}} = \frac{R_1 + R_2}{(R_1 + R_2) + R_2 A} \]

Again, for large \( A \)

\[ V_P - V_N = \frac{1}{\frac{R_2}{R_1 + R_2} A + 1} V_{in} \rightarrow 0 \text{ as } A \rightarrow \infty \]

Again we see in linear active region for large \( A \)

\[ V_P - V_N = 0 \]
Example 4  Voltage Controlled Current Source

Assume linear operation

then \( V_o = A(V_p - V_N) \)

\( V_p = V_{in} \), \( V_R = \frac{R}{R+R_L} V_o \)

\( V_o = A \left( \frac{V_{in} - \frac{R}{R+R_L} V_o}{R+R_L} \right) \)

\( V_o = A V_{in} - A \frac{R}{R+R_L} V_o \)

\( \left( \frac{AR}{R+R_L} + 1 \right) V_o = AV_{in} \)

\[ V_o = \frac{A}{\frac{AR}{R+R_L} + 1} V_{in} \]

\[ \frac{V_o}{V_{in}} = \frac{A}{\frac{AR}{R+R_L} + 1} \]

\[ i = \frac{A}{AR + (R+R_L) V_{in}} \]

\( i = \frac{1}{\frac{R}{R+R_L}} \frac{V_o}{V_{in}} \)

\[ A \rightarrow \infty \Rightarrow \frac{V_o}{V_{in}} = \frac{R+R_L}{R} \]

\[ A \rightarrow \infty \Rightarrow i = \frac{V_o}{R+R_L} \]

\[ A \rightarrow \infty \Rightarrow i = \frac{1}{\frac{R}{R+R_L}} \frac{V_o}{V_{in}} \]

\( i = \frac{1}{\frac{R}{R+R_L}} \frac{A}{V_{in}} \) independent of \( R_L \)
Example 5  Inverting Amp

\[ V_p = 0 \quad \text{node voltage method} \]

\[ \frac{V_N - V_{in}}{R_s} + \frac{V_N - V_o}{R_f} = 0 \]

\[ \left( \frac{1}{R_s} + \frac{1}{R_f} \right) \frac{V_N - V_{in} - V_o}{R_s R_f} = 0 \]

Assume operation in linear region

Then \[ V_0 = A(V_p - V_N) \quad \Rightarrow \quad V_0 = -A V_N \]

\[ V_0 = A(0 - V_N) \]

\[ \left( \frac{R_s + R_f}{R_s R_f} \right) \frac{V_N - V_{in}}{R_s} + \frac{A V_N}{R_f} = 0 \]

\[ \left( \frac{R_s + R_f}{R_s R_f} + \frac{A}{R_s} \right) \frac{V_N - V_{in}}{R_s} = 0 \]

\[ V_N = \frac{(R_s + R_f)}{R_f} \frac{V}{N} = V_{in} \left( \frac{R_s + R_f}{R_f} - \frac{R_s A}{R_f} \right) \]
\[ V_N = \frac{V_{in}}{R_s + R_f + \frac{R_s A}{R_f}} \]

\[ V_o = -A V_N = -A \frac{V_{in}}{R_s + R_f + \frac{R_s A}{R_f}} \]

With \( A \to \infty \)

\[ V_o = -A \frac{V_{in}}{R_s + \frac{R_s A}{R_f}} \]

\[ V_o = -\frac{R_f}{R_s} V_{in} \quad \text{as long as} \quad V_{EE} < -\frac{R_f}{R_s} V_{in} < V_{CC} \]

What about \( V_p - V_N \)

\[ V_p - V_N = V_{in} \left( 1 - \frac{R_s + R_f + \frac{R_s A}{R_f}}{R_f + \frac{R_s A}{R_f}} \right) \]

\[ = \left( 1 - \frac{1}{R_s + R_f + \frac{R_s A}{R_f}} \right) V_{in} \quad \text{for} \quad A = \infty \]

For very large \( A \), we can use

\[ V_p - V_N \approx 0 \quad \text{in linear regime} \]