More on Op Amp integrator

A. Understanding $v_{out}$ Graph
In the previous page $v_{in}$ is a periodic rectangular wave so its integral is a periodic triangular wave. Note that integral of $v_{in}$ over the period of $t=0$ to $1 \text{ ms}$ would be $5V \times 1\text{ms}=0.005 \text{Vs}$. But this not $v_{out}$. This is just the integral of $v_{in}$ from $t=0$ to $1 \text{ ms}$.

On the Other hand $v_{out}(t=1\text{ms}) = -\frac{1}{RC} \times \text{(integral of } v_{in} \text{ from } t=0\text{ to } 1\text{ms})$. Therefore

$$v_{out}(t=1\text{ms}) = -\frac{1}{RC} \times 0.005 \text{ with units } \frac{\text{Vs}}{\text{s}} = \text{V} \text{ which is the correct units}$$

The plot shows that $v_{out}(t=1\text{ms}) = -5 \text{V}$. This mens $\frac{1}{RC}$ must be 1000 (units= 1/s) or $RC = 1\text{ms}$

The plot also shows that $v_{out}$ returns to 0 at $t=2 \text{ms}$ and goes 5 V at $t=3 \text{ms}$.

B. Saturation
Now suppose the +supply is 4 V and the – supply is 4 volts. What would $v_{out}$ look like? We see that we have a floor of -2.5 V. So we plot that on the graph.

So the actual output would look like this:
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C. Sinusoidal Input: Steady State

Suppose a sinusoidal input \( v_{in} = V_{IN} \sin(\omega t) \). After switching transients have died, steady state will be reached. More specifically, we consider the differential equation and try solution of the form

\[
v_{out} = V_{OUT} \cos(\omega t)
\]

which anticipates a

\[
\frac{dv_{out}}{dt} = -\frac{1}{RC} v_{in} \Rightarrow \frac{dv_{out}}{dt} = -\frac{V_{IN}}{3} \sin(\omega t).
\]

For Steady State Solution we try solution of the form

\[
v_{out}(t) = V_{OUT} \cos(\omega t)
\]

and put in the diff. equ.

\[
\frac{dV_{OUT} \cos(\omega t)}{dt} = -\frac{V_{IN}}{RC} \sin(\omega t) \Rightarrow -\omega V_{OUT} \sin(\omega t) = -\frac{V_{IN}}{RC} \sin(\omega t) \Rightarrow V_{OUT} = \frac{1}{\omega RC} V_{IN}.
\]

This means, the output amplitude diminishes as the frequency increases.

\[
\frac{V_{OUT}}{V_{IN}} = \frac{1}{\omega RC} \Rightarrow \log \left( \frac{V_{OUT}}{V_{IN}} \right) = -\log(RC) - \log(\omega).
\]

Therefore in a log-log plot, we have a line with negative slope, as shown in the Figure From Sedra and Smith.