LINEARIZATION: SMALL SIGNAL MODEL OF DIODE

Diode current voltage relation is nonlinear: \( i_D = f(v_D) \)

We use calculus to expand this function at the Q-point by using the following series expansion around \( x_0 \):

\[
y = f(x) \Rightarrow f(x_0) + (x-x_0) \left. \frac{df}{dx} \right|_{x=x_0} + \frac{1}{2!} (x-x_0)^2 \left. \frac{d^2f}{dx^2} \right|_{x=x_0} + \frac{1}{3!} (x-x_0)^3 \left. \frac{d^3f}{dx^3} \right|_{x=x_0} + H.O.T.
\]

\[ H.O.T. \equiv \text{Higher Order Terms}. \]

For smooth functions and \((x-x_0)\ll 1\) we can approximate by keeping just the first two terms.

\[
y = f(x_0) + (x-x_0) \left. \frac{df}{dx} \right|_{x=x_0}. \quad \text{Linearization}
\]

This is a linear relation between \( y \) and \( x \) which is good only in the small vicinity of \( x_0 \).

Let \( y = i_D \), \( x = v_D / V_T \) and \( x_0 = V_{DQ} / V_T \). Then \( i_D = I_0 (\exp(x) - 1) \).

Let's expand this in the vicinity of the Q point.

\[
i_D = f(x_0) + (x-x_0) I_0 \exp(x) \big|_{x=x_0}. \quad \text{This approximation is valid if } (x-x_0)\ll 1 \text{ or } \frac{v_D}{V_T} - \frac{V_{DQ}}{V_T} \ll 1
\]

Substituting for \( x \) and \( x_0 \) \( \left( \text{Recall } x = v_D / V_T \text{ and } x_0 = V_{DQ} / V_T \right) \)

\[
i_D = I_0 \left[ \exp \left( \frac{V_{DQ}}{V_T} \right) - 1 \right] + \left( \frac{v_D}{V_T} - \frac{V_{DQ}}{V_T} \right) I_0 \exp \left( \frac{V_{DQ}}{V_T} \right). \quad \text{Now } I_0 \left[ \exp \left( \frac{V_{DQ}}{V_T} \right) - 1 \right] = I_{DQ} \Rightarrow
\]

\[
i_D = I_{DQ} + \frac{v_D - V_{DQ}}{V_T} (I_{DQ} + I_0). \quad \text{Now recall } v_D = V_{DQ} + v_d \text{ and } i_D = I_{DQ} + i_d
\]

Therefore \( i_D - I_{DQ} = \frac{I_{DQ} + I_0}{V_T} v_d \Rightarrow i_d = \frac{1}{r_{DQ}} v_d \)

where \( r_{DQ} = \frac{V_T}{I_{DQ} + I_0} \) is the dynamic resistance at the Q point.

To summarize:

\[
\begin{align*}
v_D & = V_{DQ} + v_d \\
i_D & = I_{DQ} + i_d \\
i_d & = \frac{1}{r_{DQ}} v_d \quad \Leftrightarrow \quad v_d = r_{DQ} i_d \quad \text{as long as } v_d \ll V_T \quad \text{with} \quad r_{DQ} = \frac{V_T}{I_{DQ} + I_0}
\end{align*}
\]

In general \( \frac{1}{r_{DQ}} = \left. \frac{\partial i_D}{\partial v_D} \right|_{Q\text{ point}} \)

Note:

When the diode deviates from being ideal, \( V_T \) is replaced by \( n V_T \).

\( n \) is called ideality factor and is usually between 1 and 2.

Larger \( n \) leads to slower current rise in forward bias.