Analysis of a Common Emitter Amplifier (Small Signal, Mid Frequency)

- The Circuit

Each circuit variable has 2 parts: 1) DC component
2) Small ac component
Objective is to find:

\[ V_{\text{out}} \] as differential voltage gain
\[ \frac{dV_{\text{out}}}{dV_{\text{in}}} \] small signal differential gain
\[ \frac{V_{\text{out}}}{V_{\text{in}}} \] small signal power gain
small signal input resistance
small signal output resistance
\[ C_3 = \text{bypass Capacitor} \]

No D.C. Source or Signal Side

\[ V_{\text{IN}} = 0 + V_{\text{in}}(t) \]
\[ V_{\text{BE}} = V_{\text{BE}} + V_{\text{be}} \]
\[ i_B = I_B + i_b(t) \]
\[ i_C = I_C + i_c(t) \]
\[ V_{\text{CE}} = V_{\text{CE}} + V_{\text{ce}} \]
\[ V_{\text{out}} = V_{\text{out}}(t) \]

\[ \text{out} \]
\[ \text{out} \]
\[ i_{\text{out}} = I_{\text{out}} + i_{\text{out}} \]

\[ C_1 \text{ and } C_2 = \text{Coupling Capacitors} \]

This is ac coupling which
Blocks DC current and
allows small signal to travel from the source to the load.
Solution
We split the problem in two parts and use superposition to find the total solution variables.

1st We focus on the DC variables to be found: $V_{BE}, I_B, I_C, V_{CE}, V_{OUT}$. For this we use the appropriate circuit for DC analysis ($V_{in}(t) = 0$).

2nd We focus on the ac variables $V_{in}(t), V_{BE}(t), I_B(t), I_C(t), V_{CE}(t)$ and $V_{out}(t), i_{in}(t), i_{out}(t)$.

**DC Analysis:** The appropriate circuit for DC analysis is obtained by letting capacitors to be open (inductors short) ac voltage sources to be short (set to 0V) (ac current sources set to open).

D.C. Analysis Circuit: The capacitance $C_2$ ensures that $V_{OUT} = 0$. 

[Diagram of the circuit with labeled components.]
We assume Circuit Parameters are chosen such that BJT is not in saturation and is in Forward Active region of operation. That is \( \beta I_B < \frac{V_{cc} - 0.2}{\frac{1}{R_c + (\frac{\beta + 1}{\beta}) R_E}} \).

\[ \beta (I_B + I_{B_{max}}) < \frac{V_{cc} - 0.2}{R_c + (\frac{\beta + 1}{\beta}) R_E} \]

Now if we choose circuit parameters such that \( \beta I_B < \frac{V_{cc} - 0.2}{R_c + (\frac{\beta + 1}{\beta}) R_E} \), this guarantees that Q point of Transistor is not in the Saturation region. Let's do this and move forward. For instance, then \( I_C = \beta I_B \), assume ideal BJT so that \( \beta \) is a constant (linear operation).

Then

\[ V_C = \frac{V_{cc}}{2} \]
Using Thevenin equivalent Circuit on the left and DC model of BJT we get:

\[ V_B = \frac{R_2}{R_1 + R_2} V_{cc} \]

\[ R_B = \frac{R_1 R_2}{R_1 + R_2} \]

\[ I_E = I_B + I_C = (\beta + 1) I_B = \frac{\beta + 1}{\beta} I_C \]

Load line:

\[ R_B I_B + V_{BE} + R_E I_B = I_B \]

\[ V_{BE} = \frac{V_B - V_{BE}}{R_B + (\beta + 1) R_E} \]

Now we see \( V_{BE} \) within a narrow range.

From this, we then use \( V_{BEQ} = V_y = \text{forward knee or forward saturation voltage} \) (usually \( 0.7 \text{V} \)).

In other words:

We ignore small variations of \( V_{BE} \) and use constant voltage drop model

\[ R I_C + V_C + R_E (I_C + I_B) = V_{cc} \]

\[ V_{CE} = V_{cc} - \left[ \frac{R_2}{R_1 + R_2} \right] I_C \]
\[ I_{BEA} = \frac{V_B - V_T}{R_B + (\beta + 1)R_E} \] or \[ V_{BEA} \text{ obtained from graphical (load line) analysis.} \]

\[ I_{cq} = \beta I_{BEA} \]

\[ V_{CEA} = V_{CC} - \left[ R_C + \frac{\beta + 1}{\beta} R_E \right] I_{cq} \]

Equation 2: DC load line relating \( I_C \) to \( V_{CE} \)

\[ I_C = \frac{V_{CC} - V_{CE}}{R_C + \left( \frac{\beta + 1}{\beta} \right) R_E} \]

Next to find \( V_{BE}, I_b, V_C, V_{CE} \) and \( V_{in}, V_{out} \)

Redraw original circuit with DC voltage source set to 0 (short) DC current source to 0 (open) Capacitors to short & inductors to open.

\[ V_{CE} = 0 \]

\[ R_E \text{ is shorted by cap} \]
Now what is small signal circuit for the transistor. Let's start with this:

\[ V_{BE} = V_{BEQ} + V_{be}, \quad i_B = I_{BQ} + i_b, \quad V_{BE} = V_{BEQ} + V_{be} \]

\[ h_c = I_{CEQ} + i_c \]

What about relation between \( i_b \) and \( V_{BE} \)?

We can replace the diode with a linearized model.
\[ V_{BE} = V_{BEQ} + V_{be}(t) \]

\[ I_B = I_{BQ} + I_b(t) \]

\[ I_b(t) = I_B - I_{BQ} \]

\[ V_{be} = V_B - V_{BEQ} \]

\[ \frac{dV_B}{dV_{be}} \]

\[ r_d = \frac{1}{r_d} \]

Dynamic resistance obtained from slope of \( I_B \) vs \( V_{BE} \) (can find it graphically)

Analytic \( I_B = \frac{I_s}{\beta} e^{\frac{V_{BE}}{V_T}} \)

\[ r_d = \frac{1}{\text{slope}} = \frac{V_T}{I_{BQ}} = \frac{I_{BQ}}{V_T} \]
\[ i_b = \left( \frac{I_{BQ}}{V_T} \right) V_b \]

but for BJT, this is called \( \frac{1}{r_i} \)

Now:

\[ V_C = \beta i_B \]

\[ I_{CQ} + I_c(+) = \beta (I_{BQ} + i_b(+) \]

but we set \( I_{CQ} = \beta I_{BQ} \) \( \Rightarrow i_c(+) = \beta i_b \)

- Small Signal model for BJT is

\[ r_d = \frac{1}{m_Q} \]

\[ r_d = \frac{V_t}{I_{BQ}} \]  

causal model

- AC Signal Analysis Circuit

\[ V_{be} = r_i i_b \]

\[ V_{be} = r_{be} i_b \]
In loop 2 \[ V_{in} = l_b r_d + (\beta + 1) l_b R_E l \]

\[ \therefore l_b = \frac{V_{in}}{r_d + (\beta + 1) R_E l} \]

\[ V_{out} = -R_L l \]

\[ \therefore V_{out} = -\frac{R_L \beta}{r_d + (\beta + 1) R_E l} V_{in} \]

\[ l_c = \beta l_b \]

\[ V_{out} = \mathbf{V}_{OUT} + V_{out} = 0 + \]

\[ \frac{d V_{out}}{d V_{in}} = \frac{d V_{out}}{d V_{in}} = \mathbf{R} = \frac{-\beta R_L l}{r_d + (\beta + 1) R_E l} \]

**Small signal voltage gain (linearized model)**

\[ \frac{d V_{out}}{d V_{in}} = \frac{V_{out}}{V_{in}} = -\frac{\beta R_L l}{r_d + (\beta + 1) R_E l} \]

\[ l_{out} = \frac{V_{out}}{R_L} \]

\[ l_{in} = l_b + \frac{V_{in}}{R_B} \]

\[ l_{in} = \mathbf{R} = \left[ \frac{1}{r_d + (\beta + 1) R_E} + \frac{1}{R_B} \right] V_{in} \]

\[ \mathbf{R}_{in} = \left[ \frac{1}{r_d + (\beta + 1) R_E} + \frac{1}{R_B} \right] V_{in} \]
\[ \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{V_{\text{out}}}{R_L} \left( \frac{1}{\frac{1}{R_c} + \frac{1}{(\beta+1)R_E}} + \frac{1}{R_B} \right) V_{\text{in}} \]

= Voltage gain \( \frac{R_{\text{in}}}{R_L} \)

Power gain = \( \frac{V_{\text{out}} V_{\text{out}}}{V_{\text{in}} V_{\text{in}}} \) = \( (\text{Voltage gain}) \frac{R_{\text{in}}}{R_c} \)

for \( R_{\text{out}} \) disconnect \( V_S \) and instead of load put \( V_{\text{test}} \)

\[ V_{\text{Test}} = V_{\text{Test}} + R_c \]

\[ V_{\text{Test}} = \frac{V_{\text{out}}}{V_{\text{test}}} = R_c \]

\[ V_{ce} + R_{E1}\frac{I_E}{2} = V_{\text{out}} \]

\[ V_{ce} = R_{E1} (\beta+1) \frac{I_E}{2} + V_{\text{out}} \]

\[ \Delta V_{ce} = - \left[ \frac{R_{E1}(\beta+1)}{1 + (\beta+1)R_E} \right] V_{\text{in}} \]

\[ = - \left[ \frac{(\beta+1)R_{E1} + \beta R_L'}{1 + (\beta+1)R_E} \right] V_{\text{in}} \]

\[ V_0 \text{ small, } \beta \text{ large} \]

\[ V_{ce} = - \left[ 1 + \frac{R_L'}{R_{E1}} \right] V_{\text{in}} \]
Example

\[ V_{cc} = 12 \text{ V} \quad R_1 = 30 \text{k}\Omega \quad R_2 = 20 \text{k}\Omega \quad R_C = 1 \text{k}\Omega \quad R_E = 2 \text{k}\Omega \]
\[ \beta = 100 \quad V_y = 0.56 \text{ V} \quad R_L = 1 \text{k}\Omega \quad R_{E1} = 1 \text{k}\Omega \quad R_{E2} = 1 \text{k}\Omega \]

Then \[ V_{BE} = \frac{20}{30+20} \times 12 = 4.8 \text{ V} \]
\[ R_B = \frac{30 \times 20 \text{(k}\Omega)^2}{(30+20) \text{ k}\Omega} = 12 \text{ k}\Omega \]

\[ I_{BE} = \frac{4.8 - 0.56}{12 + (101) \times 2} = \frac{4.24 \text{ V}}{214 \text{ k}\Omega} = 20 \mu\text{A} \]
\[ I_{cm} = \frac{1.2 - 0.2}{1 + \frac{101}{100} \times 2} = 3.58 \text{ mA} \]
\[ \beta I_B = 100 \times 20 \mu\text{A} = 2 \text{ mA} < I_{cm} \]

⇒ Forward active: \[ I_C = \beta I_B = 2 \text{ mA} \]

\[ V_{CE} = V_{cc} - \left[ R_C + \frac{R_E + 1}{\beta} \right] I_C \]
\[ = 12 - \left[ 1 + \frac{101}{100} \times 2 \right] \times 2 = 0.598 \approx 6 \text{ V} \]

\[ R_{\pi} = \frac{V_T}{I_{BE}} = \frac{0.025 \text{ V}}{20 \mu\text{A}} = 1.25 \text{ k}\Omega \]

\[ R'_L = \left[ \frac{1}{R_C} + \frac{1}{R_L} \right]^{-1} = \left[ \frac{1}{1} + \frac{1}{1} \right]^{-1} = 0.5 \text{ k}\Omega \]
Voltage Gain \( A_v \) is given by:

\[
A_v = \frac{V_{out}}{V_{in}} = \frac{-\beta R'_L}{r_n + (\beta+1)R_{E1}} = \frac{-100 \times 0.5}{102.25 + (101)1}
\]

\[= \frac{-500}{102.25} \approx -4.89
\]

Current Gain \( A_i \) is:

\[
A_i = A_v \cdot \frac{R_L'}{R_{E1}} = -4.89 \times 0.5 = -2.445
\]

Power Gain is:

\[
P_{gain} = A_v \cdot A_i \approx 13.00
\]

The expression for \( \frac{V_{ce}}{V_{in}} \) is:

\[
\frac{V_{ce}}{V_{in}} = -\left[ \frac{(\beta+1)R_{E1} + \beta R_L'}{r_n + (\beta+1)R_{E1}} \right] = -\frac{101 \times 1 + 100 \times 0.5}{1.25 + (101)1}
\]

\[= -\frac{101 + 50}{102.25} = -1.477
\]

Since \( r_n = 1.25 \ll (\beta+1)R_{E1} \),

\[
\frac{V_{ce}}{V_{in}} \approx -\left[ 1 + \frac{R_L'}{R_{E1}} \right] = \left[ 1 + \frac{0.5}{1} \right] = -1.5
\]

or \( \approx 1.5\% \) error!
Note that increasing $V_s$ → increased $I_B$ → increased $I_C$ and that $V_{CE}$ → $V_{CE1} - V' < 0$

$V_{CE2} - V_{CE1} < 0$

$(V_{CEQ} + V_{ce}) (V_{CEQ} + V_{ce1}) < 0$  $V_{CEQ}$

$\Delta V_{ce} < 0$ for $\Delta V_{in}$

$\Delta V_{ce} < 0$

$\frac{\Delta V_{ce}}{\Delta V_{in}} < 0$

$V_{ce} = -\left(\right) V_{in}$